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width is found enough for measurement with the microscope, and I have not as yet made any wider spectra for inspection without the microscope.

Expressions for the Elliptic Coordinates of a Moving Point to the Seventh Order of Small Quantities. By E. J. Stone, M.A., F.R.S., Radcliffe Observer.

The mathematical investigations of the Lunar Theory based upon the variations of the elliptic elements have been carried to the seventh order of small quantities. And this requires for completeness the use of the elliptic coordinates to the same order.

But I have failed to find the complete expressions for these coordinates to the seventh order in any of the books in the Observatory library to which I have referred.

I have, therefore, determined these expressions for my own use. And, as they have been found quite independently, they will serve as a verification of any existing results.

The notation adopted is that of Delaunay's "Théorie de la Lune."

The expressions for r, v, V, and U agree identically, to the sixth order of the small quantities, with those given by Delaunay on pages 19 and 55–59, vol. i., of the "Théorie de la Lune," and adopted by him in the formation of the general expression of the disturbing function R. But it appears that Delaunay has subsequently allowed for the terms of the seventh order in the expressions for the elliptic coordinates, with the exception of the terms multiplied by e^7 and γ^7 and the coefficients of periodical terms involving the angle $(7l)$; and he must therefore have been in possession of the complete expressions to the seventh order.

$$
\left\{
\begin{aligned}
\frac{r}{a} = {} & 1 + \frac{1}{2}e^2. \\
& + \cos l\left(-e + \frac{3}{8}e^3 - \frac{5}{192}e^5 + \frac{7}{9216}e^7\right). \\
& + \cos 2l\left(-\frac{e^2}{2} + \frac{e^4}{3} - \frac{e^6}{16}\right). \\
& + \cos 3l\left(-\frac{3}{8}e^3 + \frac{45}{128}e^5 - \frac{567}{5120}e^7\right). \\
& + \cos 4l\left(-\frac{1}{3}e^4 + \frac{2}{5}e^6\right). \\
& + \cos 5l\left(-\frac{125}{384}e^5 + \frac{4375}{9216}e^7\right). \\
& + \cos 6l\left(-\frac{27}{80}e^6\right). \\
& + \cos 7l\left(-\frac{16807}{46080}e^7\right).
\end{aligned}
\right.
$$

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$$\nu = g + l.$$

$$+\sin l\left(+2e - \frac{e^3}{4} + \frac{5}{96}e^5 + \frac{107}{4608}e^7\right).$$

$$+\sin 2l\left(+\frac{5}{4}e^2 - \frac{11}{24}e^4 + \frac{17}{192}e^6\right).$$

$$+\sin 3l\left(+\frac{13}{12}e^3 - \frac{43}{64}e^5 + \frac{95}{512}e^7\right).$$

$$+\sin 4l\left(+\frac{103}{96}e^4 - \frac{451}{480}e^6\right).$$

$$+\sin 5l\left(+\frac{1097}{960}e^5 - \frac{5957}{4608}e^7\right).$$

$$+\sin 6l\left(+\frac{1223}{960}e^6\right).$$

$$+\sin 7l\left(+\frac{47273}{32256}e^7\right).$$

$$\frac{a}{r} = 1.$$

$$+\cos l\left(+e - \frac{e^3}{8} + \frac{e^5}{192} - \frac{e^7}{9216}\right).$$

$$+\cos 2l\left(+e^2 - \frac{e^4}{3} + \frac{e^6}{24}\right).$$

$$+\cos 3l\left(+\frac{9}{8}e^3 - \frac{81}{128}e^5 + \frac{729}{5120}e^7\right).$$

$$+\cos 4l\left(+\frac{4}{3}e^4 - \frac{16}{15}e^6\right).$$

$$+\cos 5l\left(+\frac{625}{384}e^5 - \frac{15625}{9216}e^7\right).$$

$$+\cos 6l\left(+\frac{81}{40}e^6\right).$$

$$+\cos 7l\left(+\frac{117649}{46080}e^7\right).$$

$$V = h + g + l.$$

$$+\sin l\left(+2e - \frac{e^3}{4} + \frac{5}{96}e^5 + \frac{107}{4608}e^7\right).$$

$$+\sin 2l\left(+\frac{5}{4}e^2 - \frac{11}{24}e^4 + \frac{17}{192}e^6\right).$$

$$+\sin 3l\left(+\frac{13}{12}e^3 - \frac{43}{64}e^5 + \frac{95}{512}e^7\right).$$

$$+\sin 4l\left(+\frac{103}{96}e^4 - \frac{451}{480}e^6\right).$$

$$+\sin 5l\left(+\frac{1097}{960}e^5 - \frac{5957}{4608}e^7\right).$$

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$$+\sin 6l\left(+\frac{1223}{960}e^6\right).$$

$$+\sin 7l\left(+\frac{47273}{32256}e^7\right).$$

$$+\sin(2g+2l)\left[-\gamma^2-\gamma^4+4\gamma^2e^2-\gamma^6+4\gamma^4e^2-\frac{55}{16}\gamma^2e^4\right].$$

$$+\sin(2g+3l)\left[-2\gamma^2e-2\gamma^4e+\frac{27}{4}\gamma^2e^3-\frac{207}{32}\gamma^2e^5+\frac{27}{4}\gamma^4e^3-2\gamma^6e\right].$$

$$+\sin(2g+l)\left[+2\gamma^2e+2\gamma^4e-\frac{7}{4}\gamma^2e^3+\frac{5}{96}\gamma^2e^5-\frac{7}{4}\gamma^4e^3+2\gamma^6e\right].$$

$$+\sin(2g+4l)\left[-\frac{13}{4}\gamma^2e^2-\frac{13}{4}\gamma^4e^2+\frac{259}{24}\gamma^2e^4\right].$$

$$+\sin(2g)\left[-\frac{3}{4}\gamma^2e^2-\frac{3}{4}\gamma^4e^2-\frac{1}{8}\gamma^2e^4\right].$$

$$+\sin(2g+5l)\left[-\frac{59}{12}\gamma^2e^3+\frac{3221}{192}\gamma^2e^5-\frac{59}{12}\gamma^4e^3\right].$$

$$+\sin(2g-l)\left[-\frac{1}{12}\gamma^2e^3-\frac{5}{192}\gamma^2e^5-\frac{1}{12}\gamma^4e^3\right].$$

$$+\sin(2g+6l)\left[-\frac{115}{16}\gamma^2e^4\right].$$

$$+\sin(2g-2l)\left[-\frac{1}{24}\gamma^2e^4\right].$$

$$+\sin(2g+7l)\left[-\frac{9893}{960}\gamma^2e^5\right].$$

$$+\sin(2g-3l)\left[-\frac{9}{320}\gamma^2e^5\right].$$

$$+\sin(4g+4l)\left[+\frac{1}{2}\gamma^4+\gamma^6-8\gamma^4e^2\right].$$

$$+\sin(4g+5l)\left[+2\gamma^4e-\frac{85}{4}\gamma^4e^3+4\gamma^6e\right].$$

$$+\sin(4g+3l)\left[-2\gamma^4e+\frac{45}{4}\gamma^4e^3-4\gamma^6e\right].$$

$$+\sin(4g+6l)\left[+\frac{21}{4}\gamma^4e^2\right].$$

$$+\sin(4g+2l)\left[+\frac{11}{4}\gamma^4e^2\right].$$

$$+\sin(4g+7l)\left[+\frac{137}{12}\gamma^4e^3\right].$$

$$+\sin(4g+l)\left[-\frac{17}{12}\gamma^4e^3\right].$$

$$+\sin(6g+6l)\left[-\frac{1}{3}\gamma^6\right].$$

$$+\sin(6g+7l)\left[-2\gamma^6e\right].$$

$$+\sin(6g+5l)\left[+2\gamma^6e\right].$$

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$$U = \sin(g+l)\left[2\gamma - 2\gamma e^2 - \frac{1}{4}\gamma^5 + \frac{7}{32}\gamma e^4 - \frac{5}{144}\gamma e^6 + \frac{1}{4}\gamma^5 e^2 + \frac{3}{8}\gamma^7\right].$$

$$\sin(g+2l)\left[2\gamma e - \frac{5}{2}\gamma e^3 - \frac{1}{4}\gamma^5 e + \frac{17}{24}\gamma e^5\right].$$

$$\sin g\left[-2\gamma e + \frac{1}{4}\gamma^5 e\right].$$

$$\sin(g+3l)\left[+\frac{9}{4}\gamma e^2 - \frac{27}{8}\gamma e^4 + \frac{765}{512}\gamma e^6 - \frac{9}{32}\gamma^5 e^2\right].$$

$$\sin(g-l)\left[-\frac{1}{4}\gamma e^2 + \frac{1}{24}\gamma e^4 + \frac{37}{1536}\gamma e^6 + \frac{1}{32}\gamma^5 e^2\right].$$

$$\sin(g+4l)\left[+\frac{8}{3}\gamma e^3 - \frac{14}{3}\gamma e^5\right].$$

$$\sin(g-2l)\left[-\frac{1}{6}\gamma e^3 + \frac{1}{24}\gamma e^5\right].$$

$$\sin(g+5l)\left[+\frac{625}{192}\gamma e^4 - \frac{625}{96}\gamma e^6\right].$$

$$\sin(g-3l)\left[-\frac{9}{64}\gamma e^4 + \frac{9}{160}\gamma e^6\right].$$

$$\sin(g+6l)\left[+\frac{81}{20}\gamma e^5\right].$$

$$\sin(g-4l)\left[-\frac{2}{15}\gamma e^5\right].$$

$$\sin(g+7l)\left[+\frac{117649}{23040}\gamma e^6\right].$$

$$\sin(g-5l)\left[-\frac{625}{4608}\gamma e^6\right].$$

$$\sin(3g+3l)\left[-\frac{1}{3}\gamma^3 - \frac{1}{4}\gamma^5 + 3\gamma^3 e^2 - \frac{405}{64}\gamma^3 e^4 + \frac{9}{4}\gamma^5 e^2 - \frac{3}{8}\gamma^7\right].$$

$$\sin(3g+4l)\left[-\gamma^3 e - \frac{3}{4}\gamma^5 e + \frac{13}{2}\gamma^3 e^3\right].$$

$$\sin(3g+2l)\left[+\gamma^3 e + \frac{3}{4}\gamma^5 e - \frac{11}{4}\gamma^3 e^3\right].$$

$$\sin(3g+5l)\left[-\frac{17}{8}\gamma^3 e^2 + \frac{593}{48}\gamma^3 e^4 - \frac{51}{32}\gamma^5 e^2\right].$$

$$\sin(3g+l)\left[-\frac{7}{8}\gamma^3 e^2 + \frac{31}{48}\gamma^3 e^4 - \frac{21}{32}\gamma^5 e^2\right].$$

$$\sin(3g+6l)\left[-\frac{47}{12}\gamma^3 e^3\right].$$

$$\sin 3g\left[+\frac{1}{6}\gamma^3 e^3\right].$$

$$\sin(3g+7l)\left[-\frac{2567}{384}\gamma^3 e^4\right]$$

$$\sin(3g-l)\left[+\frac{5}{384}\gamma^3 e^4\right].$$

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$$\sin(5g+5l)\left[+\frac{3}{20}\gamma^5+\frac{1}{4}\gamma^7-\frac{15}{4}\gamma^5e^2\right].$$

$$\sin(5g+6l)\left[+\frac{3}{4}\gamma^5e\right].$$

$$\sin(5g+4l)\left[-\frac{3}{4}\gamma^5e\right].$$

$$\sin(5g+7l)\left[+\frac{75}{32}\gamma^5e^2\right].$$

$$\sin(5g+3l)\left[+\frac{45}{32}\gamma^5e^2\right].$$

$$\sin(7g+7l)\left[-\frac{5}{56}\gamma^7\right].$$

On the Determination of Positions of Stars for the Astrographic Catalogue at the Royal Observatory, Greenwich. By W. H. M. Christie, M.A., F.R.S., and F. W. Dyson, M.A.

Systematic measurement of the plates taken at the Royal Observatory for the Astrographic Catalogue was commenced in 1894 October. The measures were made with a glass scale in the focal plane of the viewing microscope, and since 1895 January the duplex micrometer has been almost exclusively used. A brief description of the glass scale and of the essential features of the duplex micrometer is given in the report of the Council of the R.A.S. [*Monthly Notices*, vol. lv. p. 206.] It may be mentioned that the plates, whether for the Chart or for the Catalogue, or for other objects, are all numbered consecutively in the order in which they are taken.

The coordinates of both the 6^m and 3^m images were measured, and the means were taken and used as the star's coordinates on the plate in subsequent reductions. When the images were sensibly circular the mean radius was measured; when elliptical, the greatest and least radii.

Thirty overlapping quarter-plates have been connected and compared with one another, all the stars common to the two plates being used. The stars whose places are given in the catalogues of the Astronomische Gesellschaft have been compared with the measured places on *seventy-two* plates. In both cases the linear method of reduction proposed by Professor Turner has been employed.

The chief points considered in this paper are—

1. The arrangement of the measures.
2. Connection of overlapping plates.
   (*a*) Mean discordances in the measures on overlapping plates, and